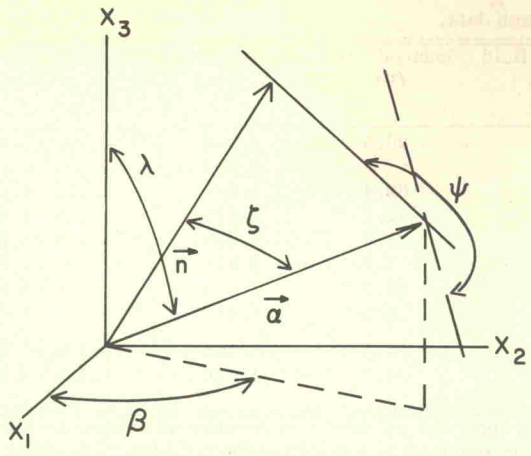

FIG. 1. Independent angular coordinates for representing strain-induced anisotropy energy. α determines magnetization axis. n determines strain axis. ψ represents rotation of plane defined by α and n about axis determined by α.

and the induced anisotropy energy

$$E_{me} = b_1 e(\alpha_1^2 n_1^2 + \alpha_2^2 n_2^2 + \alpha_3^2 n_3^2) + 2b_2 e(\alpha_1\alpha_2 n_1 n_2 + \alpha_2\alpha_3 n_2 n_3 + \alpha_3\alpha_1 n_3 n_1). \quad (2)$$

In this expression the uniaxial strain tensor $e_{ij} = e n_i n_j$ is used. The exchange and demagnetizing energies are considered implicitly in the assumption of uniform magnetization. If this assumption is correct their contribution is considerable. The crystal anisotropy is ignored. It is small in the region of strains considered.

To proceed with the averaging process, the six variables α_1, α_2, α_3, n_1, n_2, and n_3 (α and n are unit vectors) are expressed in terms of four independent angular variables as shown in Fig. 1.[21] Direction cosines are related to the angular variables by

$$\alpha_1 = \sin\lambda\cos\beta, \quad \alpha_2 = \sin\lambda\sin\beta, \quad \alpha_3 = \cos\lambda,$$

$$n_1 = \cos\xi\sin\lambda\cos\beta + \sin\xi(\cos\lambda\cos\beta\cos\psi + \sin\beta\sin\psi),$$

$$n_2 = \cos\xi\sin\lambda\cos\beta + \sin\xi(\cos\lambda\sin\beta\cos\psi - \cos\beta\sin\psi),$$

and

$$n_3 = \cos\xi\cos\lambda - \sin\xi\sin\lambda\cos\psi.$$

Assuming the polycrystal is isotropic,

$$\tfrac{1}{8}\pi^{-2}\sin\lambda\, d\lambda\, d\beta\, d\psi$$

is the probability that the magnetization lies in the range λ to $\lambda + d\lambda$ and β to $\beta + d\beta$, while the strain axis lies in a range ψ to $\psi + d\psi$. The average values of the terms appearing in the energy expression are obtained from

$$\bar{f}(\xi) = \tfrac{1}{8}\pi^{-2}\int_0^{\pi}\int_0^{2\pi}\int_0^{2\pi} f(\xi,\lambda,\beta,\psi)\sin\lambda\, d\lambda\, d\beta\, d\psi. \quad (3)$$

The resulting average induced anisotropy energy is

$$E_{me} = Be\cos^2\xi,$$

where $B = \frac{2}{5}b_1 + \frac{3}{5}b_2$, or in terms of the usual magnetostriction coefficients

$$B = -\tfrac{3}{5}[(C_{11} - C_{12})\lambda_{100} + 3C_{44}\lambda_{111}].$$

It is instructive to recall that these formulas have been derived under the assumption that the strains are identical in all grains. This assumption leads to a polycrystalline magnetostriction coefficient

$$\lambda = \frac{(C_{11} - C_{12})\lambda_{100} + 3C_{44}\lambda_{111}}{3C_{44} + C_{11} - C_{12}},$$

the fractional striction of a material along the direction in which it is magnetized. This formula may be compared with the usual polycrystalline magnetostriction formula

$$\lambda = \tfrac{2}{5}\lambda_{100} + \tfrac{3}{5}\lambda_{111},$$

derived under the assumption that each grain behaves independently mechanically. Obviously, for isotropy ($C_{11} - C_{12} = 2C_{44}$), the two formulas are identical.

The angle θ between the direction of the applied field and the direction of the magnetization is the complement of ξ. The total thermodynamic energy expression becomes

$$E = Be\sin^2\theta - H_e M_s\cos\theta. \quad (4)$$

The magnetization curve obtained from Eq. (4) for positive magnetoelastic constants is

$$M/M_s = 1, \quad H_e > -2Be/M_s$$
$$= -(M_s/2Be)H_e, \quad H_e < -2Be/M_s. \quad (5)$$

This is intermediate between the extremes obtained for the ⟨100⟩ problem and the ⟨111⟩ problem for the equivalent single-crystal behavior derived in the preceding paper.[6]

B. Independent Grain Assumption

It is quite possible that the uniform magnetization field demanded by the previous assumption does not occur. The isolated single-particle critical size within which a single domain exists for YIG is less than 1 μ. This critical size will increase for a bounded crystallite due

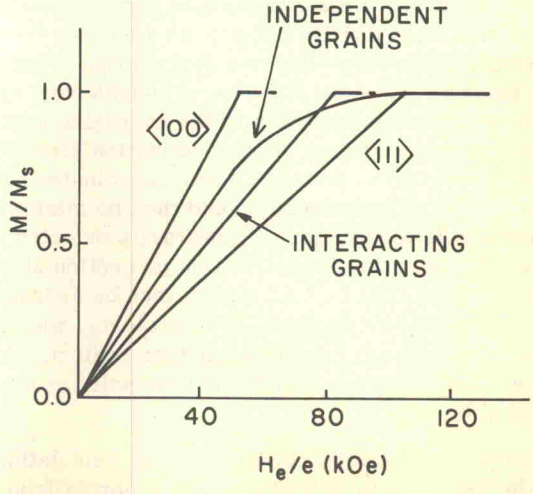

FIG. 2. Magnetization curves in polycrystalline YIG predicted from the independent and interacting grain theories. Shown for comparison are the single-crystal magnetization curves from the ⟨100⟩ and ⟨111⟩ problems.

TABLE I. Shock demagnetization data.

Shot No.	Projectile velocity (mm/μsec)	Projectile material	Mean strain in YIG	Magnetic field (Oe)	Induced[a] emf (V)	Specimen width (cm)	$\delta M/M_s$[b]
70-016	0.598	Plexiglass Rohm and Haas type-G	−0.0083	359	20.5	1.060	0.332 ± 0.066
70-030[c]	0.602			245	...	1.063	0.602 ± 0.100
70-039	0.600			258	62.4	1.067	0.515 ± 0.033
70-053	0.601			588	11.2	1.075	0.089 ± 0.034
70-057	0.596			494	21.6	1.085	0.173 ± 0.037
70-059	0.598			680	4.6	1.081	0.039 ± 0.015
71-002	0.597			421	30.3	1.023	0.260 ± 0.055
71-013	0.598			787	2.5	1.081	0.018 ± 0.010
71-015	0.551	Aluminum oxide WESGO-995	−0.0162	660	48.5	1.068	0.400 ± 0.030
71-016	0.555			935	20.5	1.032	0.173 ± 0.038

[a] This emf was developed across 10-turn pickup coils with the exception of shot No. 70-016 which used a 5-turn pickup coil.
[b] Calculated with an M_s of 128 G.
[c] On this shot, the solenoid was prematurely shorted. These values were obtained by estimating the field due to residual current and knowledge of the circuit inductances and resistances.

to a substantial decrease in surface magnet poles at the grain boundary, but not by more than an order of magnitude.[17] The grain size of the material used in the present work ranges from 5 to 25 μ. This suggests that perhaps an intragrain domain structure will nucleate in order to reduce magnetic poles which would otherwise collect heavily along grain boundaries.[17,22] This latter case holds if the energy associated with domain walls is small compared to other magnetic energies. With an intragrain domain structure there is not a uniform magnetization field as was demanded by the interacting grain assumption. In this case, it is more likely that the magnetization in each grain independently seeks a value depending only on the orientation of its crystallographic axis with the external fields.

A simple consideration shows that the independent grain assumption leads to a lower average magnetoelastic energy than the interacting grain theory. The energy from the interacting grain theory contained a part required to bring individual grains into their independent equilibrium positions and also a part required to bring these grains into collinear alignment. The latter contribution would not be present in the independent grain assumption.

The independent grain assumption is *each crystallite seeks equilibrium subject only to the requirements of the induced anisotropy field and the external magnetic field, independent of the behavior of neighboring crystallites*. A rigorous approach to the averaging procedure would be to express the magnetization direction cosines in Eq. (2) in terms of polar coordinates λ and β. The total energy expression should then be minimized with respect to λ and β for an arbitrarily oriented crystallite. The resulting magnetization projection along the direction of the applied field should then be averaged over all crystal orientations. This problem, which has been encountered previously in another context, cannot be solved explicitly for λ and β and the solution has not been completed.[14]

An alternative approach, in the spirit of calculations made by Lee,[23] is to write the average normalized magnetization

$$M/M_s=(\cos\theta)_{av}=\int F(\Omega)\cos\theta\,d\Omega/(\int F(\Omega)\,d\Omega)^{-1} \tag{6}$$

in terms of an unknown distribution function. $F(\Omega)$ is the equilibrium distribution of magnetization directions over all crystal orientations for a given applied field H_e and state of strain e. The following distribution function is assumed:

$$F(\Omega)=1, \quad \theta_1<\theta<\theta_2$$
$$=0, \quad \text{otherwise.} \tag{7}$$

The angles θ_1 and θ_2 are the extremes defined by the ⟨100⟩ problem and the ⟨111⟩ problem in the preceding paper. They are

$$\cos\theta_1=(M_s/2b_1e)H_e \tag{8}$$

and

$$\cos\theta_2=(M_s/2b_2e)H_e. \tag{9}$$

A similar distribution function has been used by Bozorth[24] in an attempt to explain low-remanence values in certain alloys and by Lee[23] to explain magnetostriction curves over the whole range of magnetization to saturation. In the work of Lee,[23] results using the uniform distribution function and a distribution function predicted theoretically by Brown[25] were compared. Experimental data strongly supported the uniform distribution function.

It must be realized that this approach will yield an approximate solution to a problem which has not, as yet, been solved exactly. The prediction is subject to the limitations of this assumption. Equation (6) becomes

$$(\cos\theta)_{av}=\int_{\theta_1}^{\theta_2}\cos\theta\sin\theta\,d\theta(\int_{\theta_1}^{\theta_2}\sin\theta\,d\theta)^{-1}$$
$$=\int_{x_2}^{x_1}x\,dx(\int_{x_2}^{x_1}dx)^{-1}\ , \tag{10}$$

where $x=\cos\theta$. A problem occurs when $\cos\theta_1$ is unity, at which point the first grains reach saturation. To freeze the upper limit of integration artificially constrains the distribution function. This problem can be circumvented by allowing the upper limit to continue but demanding that the respective contribution to $(\cos\theta)_{av}$ be unity. This gives

$$(\cos\theta)_{av}=(\int_{x_2}^{1}x\,dx+\int_{1}^{x_1}dx)(\int_{x_2}^{x_1}dx)^{-1}, \quad x_2\leq 1\leq x_1$$
$$=\int_{x_2}^{x_1}x\,dx(\int_{x_2}^{x_1}dx)^{-1}\ , \quad x_1<1. \tag{11}$$

Performing the required integration, the predicted